Advanced Studies in Pure Mathematics 12, 1987
Galois Representations and Arithmetic Algebraic Geometry
pp. 1–19

The Hyperadelic Gamma Function: A Précis

Greg W. Anderson

§ 0. Introduction

The point of departure for the results announced here is Y. Ihara's fascinating study [10] of the l-adic Tate module of the jacobian of the Fermat curve of degree l^n in the limit as $n \to \infty$. The object of this paper is to summarize some work of the author (see § 1, § 2 and § 4 below) and some joint work of the author with R. Coleman (see § 3 below), as well as to indicate the connections between our work and that of Ihara (see § 5 below).

The author's interest in these topics was originally sparked by Theorem 10 of [10] and the remark following which suggested the problem of finding a universal power series for Gauss sums into which Ihara's universal power series for Jacobi sums could be factored. We could not see how to carry out this factorization in the context of Ihara's theory, and so we developed a rather different approach to the study of the Fermat tower emphasizing relative homology groups and 1-motives, with the added feature of being adelic rather than purely l-adic. The main concepts of our theory are i) the *adelic beta function*, an object from which Ihara's universal power series for Jacobi sums can be recovered by pro-l specialization, and ii) the *hyperadelic gamma function*, an object which provides an adelic interpolation of Gauss sums and in terms of which the adelic beta function can be factored. While the main results of the theory can be formulated in classfield-theoretic terms and will be so formulated here, ideas from algebraic geometry and topology are needed to give natural proofs. The reader is referred to the forthcoming papers [1, 2] for details.

R. Coleman's interest in these topics arose in connection with a conjecture enunciated by Ihara at the Kyoto conference. Ihara conjectured a striking link between the galois-theoretic properties of the l-power torsion points of the jacobians of the l-power degree Fermat curves and the l-power roots of the numbers of the form $1-\zeta$, $1 \neq \zeta \in \mu_{l^\infty}$. (See [11] for Ihara's conjecture.) Coleman obtained a proof of Ihara's conjecture by employing his explicit reciprocity law [4], Iwasawa's theorem on local units modulo

Received April 8, 1986.