

Study of Four-Dimensional Gorenstein ASL Domains I

(Integral posets arising from triangulations of a 2-sphere)

Kei-ichi Watanabe^{*)}

To the memory of late Professor Akira Hattori

Introduction

The concept of ASL (algebras with straightening laws) plays an important role in the interaction between commutative algebra, theory of partially ordered sets and simplicial complexes. Among many examples of ASL, the most interesting ones are those which are integral domains. We are interested in the condition for a poset (a finite partially ordered set) to satisfy when there is an ASL domain R on it. This question was answered if $\dim R=2$ or $\dim R=3$ and R is Gorenstein in [8] and [3]. We call such posets integral posets.

In this article and subsequent works, we will study Gorenstein ASL domains of dimension 4, or equivalently, homogeneous coordinate rings of three-dimensional Fano varieties. As the first step for this study, we will determine the integral posets defined by attaching a minimal element T to a poset H' which is a triangulation of a 2-sphere, where we will say that a poset H is a triangulation of a topological space X if the underlying topological space of the simplicial complex $\Delta(H)$ associated to H is homeomorphic to X . Our classification is described in (2.2) and there are 18 such posets up to isomorphisms.

The concept of Cohen-Macaulay posets is defined by Cohen-Macaulay property of discrete ASL on the posets. As is shown in [8], the analogous definition of Gorenstein posets would be too strong a property for the study of Gorenstein ASL and the concept of a weakly Gorenstein poset was introduced in [8]. This property turns out to fit very well with the axiom of ASL via the theory of canonical modules.

This article is divided into 4 sections. In Section 1, we will give a characterization of a poset H' which is a triangulation of a 2-sphere by the aid of weakly Gorenstein property for H' .

In Section 2, we will review a graph-theoretical method to describe a

Received March 18, 1986.

^{*)} Partially supported by Ishida Foundation.