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Theory and Applications of Universal Linkage

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Introduction

This paper is based on a talk given at the Kyoto conference. It is in some sense a report on joint work with C. Huneke ([9]), but also contains several new results. Proofs are only included as far as they differ from the ones given in ([9]), or if they are proofs of new results.

Two proper ideals I and J in a local Gorenstein ring R are said to be *linked* (write $I \sim J$) if there is a R-regular sequence $\underline{\alpha}=\alpha_1, \cdots, \alpha_g$ in $I \cap J$ such that $J=(\underline{\alpha}): I$ and $I=(\underline{\alpha}): J$. This definition was introduced by Peskine and Szpiro who rediscovered and formalized the notion of linkage in their paper [16]. To turn linkage into an equivalence relation one considers the linkage class of an ideal I, which is the set of all R-ideals obtained from I by a finite sequence of links. We say that I is *licci* if I is in the *li*nkage *c*lass of a *c*omplete *i*ntersection ideal. It is one of the main themes in linkage theory to find necessary and sufficient conditions for two ideals to be in the same linkage class, or at least to give a characterization of licci ideals.

So far a complete solution to this problem exists only for ideals of low codimension: Let I be an ideal of grade at most two, then Apéry and Gaeta have shown that I is licci if and only if I is perfect ([1], [4]), and Hartshorne and Rao generalized this result to the non-perfect case ([17]). Moreover J. Watanabe has shown that a perfect ideal of grade 3 is licci, if R/I is Gorenstein ([22]).

While these results are the only known general sufficient conditions for two ideals to be in the same linkage class, various authors were more successful in finding necessary conditions for ideals to belong to the same linkage class, i.e., in finding properties which are invariant under linkage. First note that perfectness is preserved by linkage ([16]). As further examples for invariant properties we only mention conditions on the depth of conormal modules and Koszul homology modules ([2], [3], [5], [6], [7]). These results can be effectively used to show that certain ideals do not belong to the same linkage class ([6], [7], [14], [20]). In particular

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