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q-Version of Formulas Concerning Young Diagrams

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We give here a survey of some recent results on some formulas concerning Young diagrams and their q-analogue formulas. The following exposition divides into two parts. In Part 1, we discuss the relative version of the generating functions of ordinary plane partitions, which go back to MacMahon [4]. In Part 2, we introduce the concept of the "reverse matching", which is so to say the "surjective counterpart" of the classical (complete) matching of an incidence structure in combinatorial theory. We give an incidence structure between two finite sets by a Young diagram λ, and count the number of reverse matchings and see how its q-analogue can be constructed.

Complete proofs and other details will be published elsewhere [7, 13].

§ 1. Enumeration of ordinary plane partitions

1.1. Ordinary plane partitions

Let λ be a Young diagram.

Definition. By an "ordinary plane partition (opp) T of shape λ and largest part $\leqq r$" we mean an array of integers $0, 1, \cdots, r$ placed in each of the squares of λ, subject to the restrictions that

(1) these numbers must be non-increasing along each row;

(2) and down each column.

For example:

$$\begin{array}{rl} T= & 3\ 1\ 1\ 0\ 0. \\ & 2\ 1\ 0 \\ & 2 \\ & 0 \end{array}$$

Definition. For any given λ and r, we put

$$f_\lambda^{(r)}(q) := \sum\nolimits_T q^{|T|} \in \boldsymbol{Z}[q]$$

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