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An Algorithm to Compute the Dimensions of Algebras A and A-Modules from their Generators and Relations

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Introduction

Let k be a field and suppose an associative k-algebra A (not necessarily commutative) is given in terms of a finite set of generators and a set of finitely many defining relations among them. Moreover, suppose a left A-module M, which is finite-dimensional over k, is also given in terms of its finite generators and relations. In this note, we present an algorithm which gives the exact k-dimension of M, starting with these presentations of A and M. Not only dimensionality, our algorithm also produces explicit matrix representations of the generators of A acting on M.

If the algebra A itself is finite-dimensional over k, our algorithm can be applied to the regular A-module A to find the k-dimension of A itself and we can display the regular representation in the form of matrices.

On the other hand, so long as M is finite-dimensional, A may as well be an infinite-dimensional algebra. So, for example, the algorighm is also applicable to representations of a Lie algebra, because they can be regarded as representations of an associative algebra called the universal enveloping algebra of the Lie algebra.

One of the features of our algorithm is that it manipulates only finite-dimensional objects during all its process. Therefore it is executable on computers. In fact, we have an experience of executing this algorithm on a computer to find the dimension of an algebra defined by A. Gyoja ($H(\mathfrak{S}_4)$ in his notation, see [G]), which turned out to be 204.

The key idea in this note is that the so-called Todd-Coxeter coset enumeration method (see [T-C], [L]) in the group theory can be modified so that it could be applied to the present problem.

Notation and basic definitions

Throughout this note, let k be a field.

Let S be a set. We denote by $k\langle S\rangle$ the non-commutative polynomial

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