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Torus Embeddings and de Rham Complexes

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Introduction

An n-dimensional normal algebraic variety is said to be a torus embedding or a toric variety if it has an effective regular action of the split algebraic torus of dimension n. In [I1] and [IO], we studied reduced closed subschemes of a torus embedding which are partial unions of the orbits of the torus action, which we call *toric polyhedra* in this paper. In [I1] we gave dualizing complexes of affine toric polyhedra consisting of coherent sheaves, and as a corollary we gave criteria for the schemes to be Gorenstein or Cohen-Macaulay.

In this paper, we study the algebraic de Rham complexes of toric polyhedra. Then, we generalize the notion of toric polyhedra and define *semi-toroidal varieties* which are the varieties with singularities locally isomorphic to those of toric polyhedra in the étale topology. For the motivation to consider such varieties, see also the introduction of [IO]. Although the results in [I1] are local, the complex constructed in [I1] is generalized for semi-toroidal varieties with a good filtration, and we will show that it is a dualizing complex in a global sense.

By using this dualizing complex, we define the de Rham complex $\tilde{\Omega}^{\cdot}_X$ of a *semi-toroidal variety X with filtration.* Our de Rham complex consists of coherent sheaves and is a generalization of that of Danilov [Da], which is defined for normal varieties with toroidal singularities. For an arbitrary $\boldsymbol{C}$-scheme of finite type, du Bois [dB] defined a de Rham complex in a derived category by using the simplicial resolution of the scheme introduced by Deligne in his mixed Hodge theory [Del]. We show that our de Rham complex is equal to du Bois's for these varieties. In particular, if X is complete, the natural spectral sequence $E_1^{p,q}=H^q(X, \tilde{\Omega}^p_X) \Rightarrow H^{p+q}(X, \boldsymbol{C})$ degenerates at the E_1-terms and converges to the Hodge filteration.

Notation. For subsets A, B of a set S, we denote $A\backslash B=\{a \in A;\ a \notin B\}$. If S is an additive group, then we denote $A+B=\{a+b;\ a \in A,\ b \in B\}$.

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