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Maximal Cohen-Macaulay Modules over Gorenstein Rings and Bourbaki-Sequences

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Introduction

Motivated by recent work of Knörrer [17], Buchweitz, Greuel and Schreyer [9] who proved that a hypersurface singularity R over $\boldsymbol{C}$ is of finite Cohen-Macaulay representation type if and only if R is a simple hypersurface singularity, we were led to study, quite in general, maximal Cohen-Macaulay modules (*MCM*-modules) over Gorenstein rings. Of course, there is no deeper reason why one should restrict one's attention to Gorenstein rings. It seems, however, that representation theory over non-Gorenstein rings is fundamentally more complicated. For instance, the question of the finite Cohen-Macaulay representation type is not completely settled, though some beautiful techniques have been developed just for that purpose by M. Auslander and I. Reiten [20]. They also give two examples of non-Gorenstein rings of dim ≥ 3, which are of finite Cohen-Macaulay representation type. No other such examples are known. In [14] the first named author of the paper showed that $\boldsymbol{C}[\![x, y]\!]^G$ is of finite Cohen-Macaulay representation type, where $G \subseteq Gl(2; \boldsymbol{C})$ is finite. That these are the only 2-dimensional rings with this property was shown by Artin-Verdier [2], Auslander [3] and Esnault [13].

The great technical advantage of Gorenstein rings is that MCM-modules over Gorenstein rings are reflexive, and that the R-dual of an MCM-module is again an MCM-module.

For the rest of the paper let us always assume that (R, m) is a local Gorenstein ring. The most general question one may raise in this connection is to determine all isomorphism classes of indecomposable MCM-modules over R. Of course, this problem is posed far too generally, and should be considered only as a "Leitmotiv". The following problem seems to be more accessible: Determine all pairs of numbers (m, n) for which there exists an MCM-module M which has rank m and is minimally generated by n elements. We call (m, n) the data of M.

In [10] D. Eisenbud gives a very explicit description of the MCM-

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