Maximal Cohen-Macaulay Modules over Gorenstein Rings and Bourbaki-Sequences

Jürgen Herzog and Michael Kühl

Introduction

Motivated by recent work of Knorrer [17], Buchweitz, Greuel and Schreyer [9] who proved that a hypersurface singularity $R$ over $C$ is of finite Cohen-Macaulay representation type if and only if $R$ is a simple hypersurface singularity, we were led to study, quite in general, maximal Cohen-Macaulay modules ($MCM$-modules) over Gorenstein rings. Of course, there is no deeper reason why one should restrict one’s attention to Gorenstein rings. It seems, however, that representation theory over non-Gorenstein rings is fundamentally more complicated. For instance, the question of the finite Cohen-Macaulay representation type is not completely settled, though some beautiful techniques have been developed just for that purpose by M. Auslander and I. Reiten [20]. They also give two examples of non-Gorenstein rings of dim $\geq 3$, which are of finite Cohen-Macaulay representation type. No other such examples are known. In [14] the first named author of the paper showed that $C[[x, y]]$ is of finite Cohen-Macaulay representation type, where $G \subseteq GL(2; C)$ is finite. That these are the only 2-dimensional rings with this property was shown by Artin-Verdier [2], Auslander [3] and Esnault [13].

The great technical advantage of Gorenstein rings is that $MCM$-modules over Gorenstein rings are reflexive, and that the $R$-dual of an $MCM$-module is again an $MCM$-module.

For the rest of the paper let us always assume that $(R, m)$ is a local Gorenstein ring. The most general question one may raise in this connection is to determine all isomorphism classes of indecomposable $MCM$-modules over $R$. Of course, this problem is posed far too generally, and should be considered only as a “Leitmotiv”. The following problem seems to be more accessible: Determine all pairs of numbers $(m, n)$ for which there exists an $MCM$-module $M$ which has rank $m$ and is minimally generated by $n$ elements. We call $(m, n)$ the data of $M$.

In [10] D. Eisenbud gives a very explicit description of the $MCM$-