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# Maximal Buchsbaum Modules over Regular Local Rings and a Structure Theorem for Generalized Cohen-Macaulay Modules

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***In memory of Professor Akira Hattori***

## § 1. Introduction

The purpose of this paper is to give, applying a decomposition theorem of maximal Buchsbaum modules over regular local rings, a structure theorem for generalized Cohen-Macaulay modules relative to so-called standard systems of parameters. Before stating the results more precisely, let us recall the definition of Buchsbaum modules and generalized Cohen-Macaulay modules respectively (see (2.8) and (4.1) for a further detail).

Throughout let $A$ denote a Noetherian local ring with maximal ideal $\mathfrak{m}$ and $M$ a finitely generated $A$-module of $\dim_A M = s$. Then $M$ is said to be *Buchsbaum* (resp. *generalized Cohen-Macaulay*), if there is given a numerical invariant $I_A(M)$ of $M$ so that the equality

$$I_A(M) = \ell_A(M/\mathfrak{q}M) - e_{\mathfrak{q}}(M)$$

holds for any parameter ideal $\mathfrak{q}$ for $M$ (resp. the supremum $\sup_{\mathfrak{q}} [\ell_A(M/\mathfrak{q}M) - e_{\mathfrak{q}}(M)]$ is finite, where $\mathfrak{q}$ runs over parameter ideals for $M$, and the equality

$$I_A(M) = \sup_{\mathfrak{q}} [\ell_A(M/\mathfrak{q}M) - e_{\mathfrak{q}}(M)]$$

holds) (here $\ell_A(M/\mathfrak{q}M)$ and $e_{\mathfrak{q}}(M)$ respectively denote the length of $M/\mathfrak{q}M$ and the multiplicity of $M$ relative to $\mathfrak{q}$). This condition is equivalent to saying that any system $x_1, x_2, \cdots, x_s$ of parameters for $M$ forms a $d$-sequence on $M$ (resp. there is an integer $N \geq 1$ such that any system $x_1, x_2, \cdots, x_s$ of parameters for $M$ contained in $\mathfrak{m}^N$ forms a $d$-sequence on $M$), that is

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