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Compact Rigid Analytic Spaces
—With special regard to surfaces—

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§ 0. Introduction

The notion of rigid analytic spaces was introduced by Tate [T]. Since then the theory has been developed considerably and we now have a rather satisfactory general theory (for example, the proper mapping theorem, GAGA, resolution of singularities). Thus it may be worthwhile to see how many of the results known for compact complex manifolds have analogues for rigid analytic spaces.

In the following we shall show that many important results on compact complex manifolds have counterparts in the category of compact smooth rigid analytic spaces. For example, the notions of algebraic dimension and Kodaira dimension will be introduced and the structure theorems of algebraic reductions and pluricanonical mappings will be shown. Using these general results, we shall develop the bimeromorphic geometry of compact smooth rigid analytic spaces of dimension two (rigid analytic surfaces) which is similar to that of the complex analytic surfaces.

A rigid analytic surface appears naturally as the "generic" fibre S_η of a formal lifting of a surface S_0 in characteristic $p>0$ to characteristic zero. We shall show that the Kodaira dimensions of S_η and S_0 are equal in almost all cases. (See Theorem 5.11, below.) We conjecture that S_η and S_0 have always the same Kodaira dimension. This will be proved, if we know the structure of certain rigid analytic surfaces with algebraic dimension zero. At the moment, we have no satisfactory theory of such surfaces.

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