Advanced Studies in Pure Mathematics 10, 1987
Algebraic Geometry, Sendai, 1985
pp. 755–764

Degenerations of Surfaces

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§ 1. Introduction

In this paper, we study degenerations of surfaces with the nef canonical bundle. Here is a summary of our results. Let $\pi: \mathscr{X} \to \varDelta$ be a projective degeneration of nonsingular projective algebraic surfaces with nef canonical bundles. Then we find a modification of $\mathscr{X}$ whose canonical bundle is relatively nef.

Recently, Mori, Shokurov and Kawamata, independently, stated results more general than ours.

The proof is based on Mori's theory [5] and Kawamata's contraction theorem [2]. The remaining part of the proof is to construct a so-called flip [2], although we do not state the existence of a flip explicitly. Since this part of the proof is very elementary but might be hard to read, the readers are recommended to read this paper drawing pictures. Thanks are due to Professors M. Miyanishi, and D. Morrison for helpful advice.

§ 2. Main theorem

Let $\mathscr{X}$ be a nonsingular projective 3-fold over the complex number field $\boldsymbol{C}$ and $\varDelta$ a nonsingular curve. Suppose that we have a morphism $\pi: \mathscr{X} \to \varDelta$ satisfying the following conditions:

(i) π is surjective,

(ii) for each $p \in \varDelta$, the scheme-theoretic inverse image $\pi^{-1}(p)$ is a reduced divisor with only simple normal crossings,

(iii) if $\pi^{-1}(p)$ is nonsingular for some $p \in \varDelta$, then $\pi^{-1}(p)$ is a minimal surface, in other words, the canonical bundle of $\pi^{-1}(p)$ is nef,

(iv) the genus of $\varDelta$ is positive.

If $\pi: \mathscr{X} \to \varDelta$ (or $\mathscr{X}$ for short) satisfies these conditions, we call it an S-degeneration.

Next, we define an S-regular 3-fold as follows: Let $\mathscr{X}$ be an S-degeneration. Assume that there exists a tower of birational morphisms

Received December 9, 1985.