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Constructible Sheaves Associated to Whittaker Functions

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Introduction

Let X_0 be a proper smooth geometrically connected curve over the field $\boldsymbol{F}_q$ with q elements. Let K be the function field of X_0 over $\boldsymbol{F}_q$, $\boldsymbol{A}$ the adele ring of K, and ℓ a prime number prime to the characteristic of $\boldsymbol{F}_q$. Let $\pi_1(X_0)$ be the fundamental group of X_0. (For the fundamental group, see [8, p. 39].) We always assume that a continuous representation

$$\rho\colon\ \pi_1(X_0) \longrightarrow \mathrm{GL}(n, \bar{\boldsymbol{Q}}_\ell) \qquad (\bar{\boldsymbol{Q}}_\ell\colon \text{an algebraic closure of } \boldsymbol{Q}_\ell)$$

of $\pi_1(X_0)$ factors through

$$\rho\colon\ \pi_1(X_0) \longrightarrow \mathrm{GL}(n, E),$$

where E is a finite extension of $\boldsymbol{Q}_\ell$.

Such a ρ gives rise to an L-function

$$L(\rho, s) = \prod_{v \in |X_0|} \det(1 - \mathrm{Nm}(v)^{-s}\rho(\mathrm{Fr}_v))^{-1} \in \bar{\boldsymbol{Q}}_\ell[[q^{-s}]],$$

where $|X_0|$ is the set of closed points of X_0, and Fr_v is the geometric Frobenius substitution at v.

Langlands ([6, p. 211]) asked whether it is an automorphic L-function. (For the definition of automorphic L-function, see [2, p. 49]). Drinfeld (cf. [3]) has solved this problem for $n=2$. First he expressed the Whittaker function associated to ρ by the trace of the Frobenius substitution on some constructible sheaf. Next, he proved geometrically that the Shalika transform (cf. [9]) of the Whittaker function turns out to be an automorphic form.

For a representation ρ as above, we can associate a function f on $\mathrm{GL}(n, \boldsymbol{A})$ called the Whittaker function for ρ. By the functional equation satisfied by the Whittaker function, it can be regarded as a function on $U_K\backslash \mathrm{GL}(n, \boldsymbol{A})/\mathrm{GL}(n, \hat{O})$, where U_K is the subgroup of upper triangular

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