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Algebraic Cycles on Hypersurfaces in P^N

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Dedicated to Professor Masayoshi Nagata on his sixtieth birthday

§ 0. Introduction

The main theme of this paper is to exploit an idea that the Abel-Jacobi map can be studied in close analogy with the cycle map as far as the behavior of the image is concerned.

Given a nonsingular complex projective variety X, let $CH^r(X)$ be the Chow group of codimension r algebraic cycles on X modulo rational equivalence ($0 \leq r \leq \dim X$). The cycle map is a homomorphism

$$\gamma^r\colon\ CH^r(X) \longrightarrow H^{2r}(X, \boldsymbol{Z}) \simeq H_{2(n-r)}(X, \boldsymbol{Z})$$

and the Abel-Jacobi map is a homomorphism

$$\psi^r\colon\ CH^r(X)_{\mathrm{hom}} \overset{\mathrm{def}}{=} \mathrm{Ker}\,(\gamma^r) \longrightarrow J^r(X)$$

where $J^r(X)$ is the r-th intermediate Jacobian of X. Set

$$\mathscr{C}^r(X) = \mathrm{Im}\,(\gamma^r), \qquad J^r_h(X) = \mathrm{Im}\,(\psi^r).$$

Further, letting $CH^r(X)_{\mathrm{alg}}$ denote the subgroup of $CH^r(X)$ of algebraic cycles algebraically equivalent to zero, we set

$$J^r_a(X) = \psi^r(CH^r(X)_{\mathrm{alg}}).$$

It is known that $J^r_a(X)$ is a complex subtorus of $J^r(X)$ having structure of abelian variety and that the quotient group $J^r_h(X)/J^r_a(X)$ is at most countable.

Now we propose to study the behavior of $J^r_a(X)$ for X hypersurfaces of dimension $2r-1$ in a projective space in comparison with that of $\mathscr{C}^r(X')$ for X' hypersurfaces of dimension $2r$. Such an idea must have been known for some time, but personally it has occurred to us in trying to understand Griffiths' theorem on the image of the Abel-Jacobi map for generic hypersurfaces of odd dimension ([G3]). We see that there is a strong analogy between Griffiths' result and the classical theorem of Max Noether. Inspired by this, we have recently found some explicit example of hypersurfaces defined over $\boldsymbol{Q}$ (the field of rational numbers) such that $J^r_a(X) = 0$

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