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K. Saito's Period Map for Holomorphic Functions with Isolated Critical Points

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Dedicated to Professor Masayoshi Nagata on his sixtieth birthday

Introduction

The purpose of this survey is to explain the theory of Kyoji Saito on the period map for the universal unfolding of a holomorphic function with only isolated critical points, which he has been developing in a series of published and unpublished papers [SK1] through [SK19]. We can take advantage of the added perspective of algebraic analysis developed by Sato, Kashiwara, Kawai and Mebkhout as well as results obtained by Morihiko Saito.

For a positive integer n which we eventually assume to be even, let $F_1(x, t')$ be a holomorphic function on a neighborhood X of the origin in $\boldsymbol{C}^{n+1}\times\boldsymbol{C}^{\mu-1}$ with coordinates $(x, t')=(x_0, \cdots, x_n, t_2, \cdots, t_\mu)$ such that $F_1(0, 0)=0$ and that the function $F_1(x, t')$ in $x\in\boldsymbol{C}^{n+1}$ for each fixed t' has at most isolated critical points. Consider the holomorphic map

$$\varphi\colon X\longrightarrow S\subset\boldsymbol{C}\times T\subset\boldsymbol{C}\times\boldsymbol{C}^{\mu-1},$$

defined by $\varphi(x, t')=(F_1(x, t'), t')$ for a neighborhood S (resp. T) of the origin in $\boldsymbol{C}\times\boldsymbol{C}^{\mu-1}$ (resp. $\boldsymbol{C}^{\mu-1}$) with coordinates $s=(t_1, t')$ (resp. $t'=(t_2, \cdots, t_\mu)$). Thus φ gives rise to a family, parametrized by $s=(t_1, t')\in S$, of germs of n-dimensional hypersurfaces

$$X_s:=\varphi^{-1}(s)=\{(x, t')\in X;\ t_1-F_1(x, t')=0\}$$

in $\boldsymbol{C}^{n+1}$ with at most isolated singular points.

The *critical locus*

$$C:=\mathrm{Specan}\,(\mathcal{O}_X/(\partial F_1/\partial x_0, \cdots, \partial F_1/\partial x_n))$$

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