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Fourier Functor and its Application to the Moduli of Bundles on an Abelian Variety

Shigeru Mukai

At the symposium we talked on the vector bundles on a $K3$ surface and applications to the geometry of a $K3$ surface. Most content of our talk is contained in the paper "On the moduli space of bundles on $K3$ surfaces, I" to appear in the proceeding of the symposium on vector bundles at Tata Institute in 1984. In this article we discuss the vector bundles on an abelian variety instead.

In [12], we have defined the Fourier functor and shown its basic properties. This functor is a powerful tool for investigating the vector bundle (or coherent sheaves, more generally) on an abelian variety as we have shown for the Picard bundles in [12]. In this article, generalizing the results in [12], we shall show that a sheaf and its Fourier transform have the same local (in the Zariski topology) moduli space and apply this to the study of the moduli space of vector bundles on an abelian variety X. In Section 1, we shall show that the moduli space of the Picard bundles is non-reduced in the case X is the Jacobian variety of a hyperelliptic curve of genus≥ 3. In the remaining sections, we shall mainly study the sheaves of U-type, which were first studied in [20] over an abelian surface.

Definition 0.1. Let (X, ℓ) be a principally polarized abelian variety, *i.e.*, ℓ is an algebraic equivalence class of ample line bundles with Euler Poincaré characteristic 1. A sheaf E on X is *of U-type* if there exists a homomorphism $f\colon L^{-1}\to H$ from a line bundle L^{-1} in the class $-\ell$ to a homogeneous vector bundle H such that $\mathrm{Hom}(f, P)\colon \mathrm{Hom}_{\mathcal{O}_X}(H, P)\to \mathrm{Hom}_{\mathcal{O}_X}(L^{-1}, P)$ is injective for every $P\in \mathrm{Pic}^0 X$ and E is isomorphic to the cokernel of f. (A vector bundle H is homogeneous if and only if there exists a filtration $0=H_0\subset H_1\subset H_2\subset\cdots\subset H_n=H$ such that $H_i/H_{i-1}\in \mathrm{Pic}^0 X$ for every $i=1, \cdots, n$, cf. Theorem 4.17 in [11] and Section 3 in [12].)

We shall show in Section 2 that a sheaf of U-type is simple and the isomorphism classes of rank r sheaves of U-type form an open subset isomorphic to $X\times \mathrm{Hilb}^{r+1} X$ in the moduli space of simple sheaves, where $\mathrm{Hilb}^{r+1} X$ is the Hilbert scheme of 0-dimensional subschemes of length

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