Advanced Studies in Pure Mathematics 10, 1987
Algebraic Geometry, Sendai, 1985
pp. 449–476

# The Chern Classes and Kodaira Dimension of a Minimal Variety

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## § 1. Introduction

This paper deals with a sort of inequality for the first and second Chern classes of normal projective varieties with numerically effective canonical classes (Theorem 1.1); to some extent it is a continuation of the author's previous paper [Mil] in which the surface case was discussed. Our generalized inequality will be, however, farther-reaching in connexion with the classification theory of algebraic varieties developed by S. Iitaka, K. Ueno, M. Reid, E. Viehweg, S. Mori, Y. Kawamata and many others. For instance, we can derive the non-negativity of the Kodaira dimension for certain "*minimal*" threefolds (Theorem 1.2), which is a crucial step in the classification of threefolds after the construction of minimal models of non-uniruled varieties (the so-called "minimal model conjecture", see (6.5) below).

The precise statements of our results are as follows:

**Theorem 1.1** (characteristic 0). *Let $k$ be an algebraically closed field of characteristic* 0 *and $X$ a normal projective $\boldsymbol{Q}$-Gorenstein variety of dimension $n \geqq 2$ over $k$ with singular locus of codimension $\geqq 3$. Assume that the canonical divisor $K_X \in \operatorname{Pic}(X) \otimes \boldsymbol{Q}$ is numerically effective. Let $\rho\colon Y \to X$ be any resolution of the singularities. Then, for arbitrary ample divisors $H_1, \cdots, H_{n-2}$ on $X$, the inequality*

$$(3c_2(Y) - c_1^2(Y))(\rho^* H_1 \cdots \rho^* H_{n-2}) \geqq 0$$

*holds. In particular, if $n=3$, the* 1*-cycle $\rho_*(3c_2(Y) - c_1^2(Y))$ is pseudo-effective, i.e., its numerical class is a limit of those of effective rational* 1*-cycles.*

**Theorem 1.2** (characteristic 0). *Let $X$ be a normal projective threefold with only canonical singularities. Assume that the canonical divisor $K_X \in \operatorname{Pic}(X)_{\boldsymbol{Q}}$ is numerically effective. If $X$ is Gorenstein or $K_X^2$ is numerically*

Received November 21, 1985.