Advanced Studies in Pure Mathematics 10, 1987
Algebraic Geometry
pp. 399–414

The Rationality of the Moduli Spaces of Vector Bundles of Rank 2 on P^2
(with an appendix by Isao Naruki)

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Introduction

Let k be an algebraically closed field and $M(c_1, c_2)$ the moduli space of vector bundles E of rank 2 on P^2_k with $c_1(E)=c_1$ and $c_2(E)=c_2$. Through tensoring a suitable line bundle, $M(c_1, c_2)$ is isomorphic to one of $M(0, a)$ or $M(1, b)$. It is known that

1) $M(0, a)=\phi$ unless $a\geqq 2$ while $M(1, b)=\phi$ unless $b\geqq 1$,

2) $M(0, a)$ and $M(1, b)$ are non-singular, irreducible, quasi-projective varieties for all $a\geqq 2$ and all $b\geqq 1$,

3) all the $M(0, a)$ and $M(1, b)$ are unirational.

W. Barth [1] stated that $M(0, a)$ is rational for every $a\geqq 2$ while the rationality of $M(1, b)$ was proved by K. Hulek [4]. Recently we found a serious gap in the proof of Barth. We come upon a similar gap in Hulek's proof though it can be fortunately corrected in an obvious way (see the footnote on p. 266 of [4]). On the other hand, G. Ellingsrud and S.A. Strømme [3] showed the following results by a method completely different from Barth's and Hulek's.

Theorem 0.1. (1) *$M(1, b)$ is rational for every $b\geqq 1$.*

(2) *$M(0, a)$ is rational if $a\geqq 2$ and a is odd.*

(3) *If a is even and $a\geqq 2$, then there is a P^1-bundle in the étale topology* (*see Remark* 3.8) *over a dense open set of $M(0, a)$ which is rational.*

If we try to fix the proof of Barth, then we encounter the problem of rationality of the quotient of the affine cone over a Grassmann variety by an action of a finite group which is a semi-direct product of S_n by $(Z/2Z)^{\oplus n}$. The situation is going to be explained in Section 1. Combining the above theorem, Theorem 7.17 of [7], Theorem 2 of [5] and Theorem 3.17 of [8], we see that $M(c_1, c_2)$ is rational if it is fine. We shall give a

Received September 21, 1985.
Revised April 16, 1986.