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Stability of the Pluricanonical Maps of Threefolds

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Introduction

Let X be a non-singular projective variety with the non-negative Kodaira dimension $\kappa(X)$ defined over the field of complex numbers.

Letting K_X to be a canonical divisor of X, we denote by $\Phi_{|nK_X|}$ the rational map associated with the linear system $|nK_X|$. Then there exists some positive integer n such that dim (Im $\Phi_{|nK_X|}$) $=\kappa(X)$ and that the generic fiber of $\Phi_{|nK_X|}: X \cdots\to \operatorname{Im} \Phi_{|nK_X|}$ is geometrically irreducible. Such a fibration is called the stable canonical map or the Iitaka fibration.

In this paper we consider the following problem:

Let X be a non-singular projective variety with $\kappa(X)>0$. For which value of n, does $|nK_X|$ define the stable canonical map?

For surfaces, this problem was studied in detail by Bombieri [2], Kodaira [17] and Iitaka [6]. For non-singular threefolds of general type with K_X numerically effective, Wilson [27], Benveniste [1] and Matsuki [21] treated the problem. Recently Kollár [18] investigated threefolds with irregularity $\geqq 4$.

The purpose of the present paper is to generalize their results.

In Section 3, we study the birationality of the pluricanonical maps of threefolds of general type. Our result is stated as follows:

Theorem (3.4). *Let X be a non-singular threefold of general type which has a minimal model of index r. Then $\Phi_{|nK_X|}$ is a birational map for $n \geqq n_0$ where*

$$
\begin{aligned}
n_0 &= 9 && \text{if } r=1,\\
n_0 &= 13 && \text{if } r=2,\\
n_0 &= 4r+4 && \text{if } 3\leqq r\leqq 5,\\
n_0 &= 4r+3 && \text{if } r\geqq 6.
\end{aligned}
$$

For the definition of the minimal model and its index, see Section 1. Note that the index is independent of the choice of a minimal model.

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