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Coverings of Algebraic Varieties

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Introduction

In this article we survey and prove some of the results about (unramified) coverings of algebraic varieties. Recently Madhav Nori has asked the following question.

Conjecture A. *Let S be a projective, non-singular surface over the field of complex numbers $\boldsymbol{C}$. Suppose D is an effective divisor on S with $D^2>0$. Let N be the normal subgroup of $\pi_1(S)$ generated by the images of the fundamental groups of the non-singular models of all the irreducible components of D. Then the index $[\pi_1(S):N]$ is finite.*

If the conjecture is true, then any surface (smooth, projective) possessing a (possibly singular) rational curve of positive self-intersection would have a finite fundamental group! In [7] Nori verifies the conjecture in a special case. Surprisingly, this conjecture is related to the following old question:

Conjecture B. *Let X be a smooth, projective variety over $\boldsymbol{C}$. Then the universal covering space of X is holomorphically convex.*

Recall that a complex manifold M is said to be holomorphically convex if given a sequence of distinct points $x_1, x_2, \cdots$ in M without a limit point in M, there exists a holomorphic function f on M such that the set $\{f(x_n)\}_{n=1,2,\dots}$ is unbounded.

A compact, complex manifold is vacuously holomorphically convex. We will prove the following results in this paper.

(1) (*See* § 1, *Proposition* 1). *Suppose every covering space $\tilde{S}\to S$ is holomorphically convex. Then Conjecture* A *is valid for S, if D is an irreducible curve. If the universal covering space is holomorphically convex, then Conjecture* A *is true, if D is a rational curve (possibly singular).*

(2) (*See* § 1, *Theorem*). *Let $\pi: S\to\Delta$ be an elliptic surface. If*

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