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On Polarized Manifolds Whose Adjoint Bundles Are Not Semipositive

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Introduction

Let L be an ample (not necessarily very ample) line bundle on a projective variety M with $\dim M=n$ having only rational normal Gorenstein singularities. Let ω be the dualizing sheaf and let K be the line bundle such that $\mathcal{O}_M(K)=\omega$. We will study the line bundles $K+tL$, where t is a positive integer. By the base point free theorem (cf. [K2; Theorem 2.6]), we have $\mathrm{Bs}|m(K+tL)|=\emptyset$ for $m\gg 0$ if $K+tL$ is *numerically semipositive* (=*nef*, for short), which means $(K+tL)C\geqq 0$ for any curve C in M. We do not know, however, how large m should be. Here we just pose the following:

Conjecture. $\mathrm{Bs}|m(K+tL)|=\emptyset$ *if* $m>n+1-t$ *and if* $K+tL$ *is nef.*

In this paper we will study the case in which $K+tL$ is not nef. Our result is similar to those in [M1] and is based on the theory in [K2], [KMM]. We use also techniques in [M1] and [M2].

Basically we use the customary notation in algebraic geometry. Line bundles and the invertible sheaves of their sections are used interchangeably. Tensor products of them are denoted additively while we use multiplicative notation for intersection products in Chow rings. The pull-back of a line bundle B on V by a morphism $h\colon T\to V$ is denoted by B_T, or often just by B when confusion is impossible or harmless.

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After the first version of this note was completed, I received an interesting preprint [Io] of P. Ionescu. There he independently obtained our results Theorems 1, 2 and 3′ together with Corollaries 1 and 2 although he assumed that M is non-singular. His method is different from ours and is

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