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Complete Intersections with Growth Conditions

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Introduction

Cornalba-Griffiths [1] posed the following problem: Let $X \subset \boldsymbol{C}^3$ be a smooth algebraic curve. Do there exist two holomorphic functions f, g of finite order on $\boldsymbol{C}^3$ such that X is the complete intersection of the surfaces $\{f=0\}$ and $\{g=0\}$, i.e., f and g generate the ideal of X? By Serre [6] one knows that it is not always possible to find two polynomials with this property. In fact, it follows from the solution of the Serre conjecture (cf. Quillen [5] and Suslin [8]) that X is an algebraic complete intersection if and only if the canonical bundle of X is algebraically trivial (the same is true more generally for two-codimensional algebraic submanifolds of $\boldsymbol{C}^n$). On the other hand, one knows [2] that any smooth analytic curve X in a Stein manifold M of dimension $\leqslant 3$ is analytically a complete intersection.

The purpose of the present paper is to solve the problem of Cornalba-Griffiths. In fact we prove a more general theorem: Let $X \subset \boldsymbol{C}^n$ be an algebraic submanifold of pure codimension two such that the canonical bundle of X is topologically trivial. Then the ideal of X is generated by two entire functions of finite order (cf. Corollary 3.2). Note that the condition on the canonical bundle is necessary, since the normal bundle of every complete intersection is trivial. If X is a curve, this condition is automatically fulfilled, since on an open Riemann surface every holomorphic vector bundle is analytically (a fortiori topologically) trivial.

The proof uses analytic and algebraic methods. As an analytic tool we prove an extension and division theorem with growth conditions (cf. § 2). With the help of this theorem the problem is algebraically reduced to the application of a theorem of Quillen-Suslin on projective modules over a polynomial ring $B[T]$ and in this application B is the ring of certain functions of finite order.

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