

Uniqueness of Einstein Kähler Metrics Modulo Connected Group Actions

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Dedicated to Professor Kunihiko Kodaira on his seventieth birthday

§0. Introduction

Throughout this paper, we fix an arbitrary n -dimensional compact complex manifold X with positive first Chern class $c_1(X)_R > 0$. We then put

\mathcal{K} : the set of all Kähler forms on X representing $2\pi c_1(X)_R$,
 $\mathcal{K}^+ := \{\omega \in \mathcal{K} \mid \omega \text{ has positive definite Ricci tensor}\}$,
 $\mathcal{E} := \{\omega \in \mathcal{K} \mid \omega \text{ is an Einstein form}\}$,
 $C^\infty(X)_R$: the space of real-valued C^∞ -functions on X ,
 $\text{Aut}(X)$: the group of holomorphic automorphisms of X ,
 $G := \text{Aut}^0(X)$: the identity component of $\text{Aut}(X)$.

Furthermore, $\text{Aut}(X)$ is always assumed to act from the right on \mathcal{K} by $(\omega, g) \in \mathcal{K} \times \text{Aut}(X) \mapsto g^*\omega \in \mathcal{K}$.

The main purpose of this paper is to prove the uniqueness of Einstein Kähler metrics, if any, on X up to G -action. Such uniqueness was known only for i) Kähler C -spaces (cf. Matsushima [12]) and ii) some non-homogeneous Einstein manifolds recently discovered by Sakane [13]. Now, the correct statement we obtain has the following stronger form as announced earlier in [9]:

Theorem A. Fix an element ω_1 of \mathcal{K} . Let $\mu^+ : \mathcal{K}^+ \rightarrow \mathbf{R}$ be the restriction to \mathcal{K}^+ of the \mathcal{K} -energy map $\omega \in \mathcal{K} \mapsto M(\omega_1, \omega) \in \mathbf{R}$ of the Kähler manifold (X, ω_1) (see Section 1, also [9]). Assume that $\mathcal{E} \neq \emptyset$. Then

(i) μ^+ is bounded from below and takes its absolute minimum exactly on \mathcal{E} .

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