Advanced Studies in Pure Mathematics 10, 1987
Algebraic Geometry, Sendai, 1985
pp. 1–10

Plurícanonical Systems of Algebraic Varieties of General Type of Dimension $\leqq 5$

Tetsuya Ando

§ 1. Introduction

Let X be a non-singular projective variety of dimension n over an algebraically closed field k of characteristic zero.

Definition. A Cartier divisor D is called *nef and big* if
(i) $(D \cdot C)_X \geqq 0$ for every curve C on X, and
(ii) $D^n > 0$.
In this paper we consider the following problem:

Assume that the canonical divisor K_X is nef and big. Then, find the small integer $m(n)$ depending only on dim X such that the rational map $\Phi_{|mK_X|}$ associated with $|mK_X|$ is a birational map onto its image for every $m \geqq m(n)$.

The following result is one of the partial answers.

Theorem 1. *Assume that the canonical divisor K_X is nef and big, and that $n \leqq 5$. Then there exists a positive integer $m(n)$ such that for every $m \geqq m(n)$, the rational map $\Phi_{|mK_X|}$ associated with $|mK_X|$ is a birational map onto its image. Here $m(n)$ is given as follows:*

$$m(1)=3, \quad m(2)=5, \quad m(3)=8, \quad m(4)=20, \quad m(5)=36.$$

This theorem will be proved in Section 2. Moreover, in Section 3, we will show that improved results will be obtained if we use Miyaoka's inequality explained later. In Section 4, a similar theorem is shown to hold for the anti-pluricanonical systems.

We now recall known results concerning our problem. Matsusaka first proved the existence of $m(n)$ ([8], [9], [5], [4]). Maehara presented a function $m(n)$ for general n ([6]). Wilson gave $m(3)=25$ ([12]). Recently, Benveniste presented $m(3)=8$ ([1]). Matsuki improved his argument and finally showed $m(3)=7$ ([7]). Our result concerns only the values $m(4)$

Received June 26, 1985.
Revised November 30, 1985.