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A Characterization of the Kahn-Priddy Map

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Dedicated to Professor Nobuo Shimada on his 60th birthday

§ 1. Introduction and statements of main results

We denote by P^n the real n-dimensional projective space. E^kP^n for $k\geq 0$ denotes the k-fold reduced suspension of P^n and $E^k\phi_n$: $E^{n+k}P^{n-1}\to S^{n+k}$ the k-fold reduced suspension of a mapping ϕ_n: $E^nP^{n-1}\to S^n$. $E^k\phi_n$ for $n\geq 2$ is called a Kahn-Priddy map if the homotopy class of the restriction $E^k\phi_n|S^{n+k+1}$ generates $\pi_{n+k+1}(S^{n+k})$. We denote by $s(n)$ the number of i such that $0<i\leq n$ and $i\equiv 0$, 1, 2 or 4 mod 8.

By abuse of notation, we often use the same letter for a mapping and its homotopy class. Our first result is the following

Theorem 1.1. *Let ϕ_{2n+1}: $E^{2n+1}P^{2n}\to S^{2n+1}$ be a Kahn-Priddy map. Then the order of $E^k\phi_{2n+1}$ is $2^{s(2n)}$ for $k\geq 0$.*

For a CW-complex K, we put $\pi^n(K)=[K, S^n]$ which is n-th cohomotopy group if $K=EK'$ or dim $K\leq 2n-2$. Let H: $\pi^n(E^nP^{n-1})\to \pi^{2n-1}(E^nP^{n-1})$ be the Hopf homomorphism [10] and p_n: $P^n\to S^n$ the canonical map. Then our second result is the following

Theorem 1.2. *ϕ_{2n+1}: $E^{2n+1}P^{2n}\to S^{2n+1}$ is a Kahn-Priddy map if and only if $H(\phi_{2n+1})=E^{2n+1}p_{2n}$.*

Our basic idea is based on [3]. To prove Theorem 1.1, we shall use the $\widetilde{KO}$-group of P^n [1] and the suspension order of the identity class of $E^{2n}P^{2n}$ [9]. To prove Theorem 1.2, we shall use the essential uniqueness of Kahn-Priddy maps [2] and the EHP-sequence.

The problem determining the order of the Kahn-Priddy map was posed by Goro Nishida who solved it in the case of odd primes [7]. The

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