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On the Orders of the Generators in the 18-Stem of the Homotopy Groups of Spheres

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§ 1. Introduction

Let π_i^n be the 2-component of $\pi_i(S^n)$. The purpose of this paper is to determine the orders of the generators of the groups π_{n+18}^n for $n=10$, 11 and 12. H. Toda determined π_{n+i}^n for $i\leqq 19$ and all n in [2]. He defined the generators λ'', ξ'' of π_{28}^{10} and λ', ξ' of π_{29}^{11}, making use of Propositions in [2, Chapter 11] which assert the existence of new generators under certain conditions. Thus he obtained the group structures and generators of π_{n+18}^n ($n=10$, 11 and 12) in [2, Theorem 12.22], which states

$$\pi_{28}^{10}\approx Z_8\oplus Z_2\oplus Z_2: \textit{ generated by } \lambda'', \xi'' \textit{ and } \eta_{10}\circ\bar{\mu}_{11},$$

$$\pi_{29}^{11}\approx Z_8\oplus Z_4\oplus Z_2: \textit{ generated by } \lambda', \xi' \textit{ and } \eta_{11}\circ\bar{\mu}_{12},$$

$$\pi_{30}^{12}\approx Z_{32}\oplus Z_4\oplus Z_4\oplus Z_2: \textit{ generated by } \xi_{12}, E\lambda', E\xi' \textit{ and } \eta_{12}\circ\bar{\mu}_{13}.$$

But the orders of λ'', ξ'', λ' and ξ' were not determined in [2]. In this paper, in order to investigate their properties further, we shall define new elements $\bar{\bar{\xi}}''$, $\bar{\bar{\lambda}}''$ and $\tilde{\lambda}''$ of π_{28}^{10} by Toda brackets. Then making use of the various properties of Toda brackets, we shall obtain many relations involving these new elements and the ones defined in [2]. These results will be stated in Propositions 1–4: These relations enable us to determine the orders of λ'', ξ'', λ' and ξ'. As the main results of this paper we shall determine the direct summands of π_{n+18}^n for $n=10$, 11 and 12;

Theorem. *The group π_{n+18}^n ($n=10$, 11 and 12) has the following direct summands with the generators defined by H. Toda in* [2].

$$\pi_{28}^{10}=Z_8\{\xi''\}\oplus Z_2\{\xi''\pm\lambda''\}\oplus Z_2\{\eta_{10}\circ\bar{\mu}_{11}\},$$

$$\pi_{29}^{11}=Z_8\{\xi'\}\oplus Z_4\{\xi'+\lambda'\}\oplus Z_2\{\eta_{11}\circ\bar{\mu}_{12}\},$$

$$\pi_{30}^{12}=Z_{32}\{\xi_{12}\}\oplus Z_4\{E\xi'+4\xi_{12}\}\oplus Z_4\{E\xi'+E\lambda'\}\oplus Z_2\{\eta_{12}\circ\bar{\mu}_{13}\},$$

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