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Non-free-periodicity of Amphicheiral Hyperbolic Knots

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A knot K in the 3-sphere S^3 is said to have *free period* n if there is an orientation-preserving homeomorphism f on S^3 such that

(1) $f(K)=K$,

(2) f is a periodic map of period n,

(3) $\mathrm{Fix}(f^i)=\phi$ $(1 \leqq i \leqq n-1)$.

Hartley [3] has given very effective methods for determining the free periods of a knot, and has identified the free periods of all prime knots with 10 crossings or less with eight exceptions. Since then, Boileau [1] has calculated the symmetry groups of the "large" Montesinos knots, and has shown that four of the rest have no free periods. The remaining knots are 8_{10}, 8_{20}, 10_{99} and 10_{123} (cf. [5]). By Hartly-Kawauchi [4], 10_{99} and 10_{123} are the only prime knots with 10 crossings or less which are strongly positive amphicheiral. Moreover, it follows from the Theorem of [4] that the polynomial condition given by [3] (Theorem 1.2) does not work for determining whether a strongly positive amphicheiral knot has free period 2 or not.

The purpose of this paper is to prove the following theorem:

Theorem. *Any amphicheiral hyperbolic knot has no free periods.*

In particular, 10_{99} and 10_{123} have no free periods. A circumstantial evidence for this theorem is given by the non-trivial torus knots, which have infinitely many free periods and are not amphicheiral.

§ 1. Some lemmas

Let K be a knot in S^3 which has free period n, and f be a periodic map on S^3 realizing the free period n. Let N be an equivariant tubular neighbourhood of K and put $E=\mathring{S}^3-N$.

Lemma 1. *K does not have an f-invariant longitude curve. That is, $f(l)\neq l$, for any simple loop l in ∂N such that $l\sim K$ in $H_1(N)$ and $l\sim 0$ in $H_1(E)$.*

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