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On the Signature Invariants of Infinite Cyclic Coverings of Even Dimensional Manifolds

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§ 0. Introduction

We consider a compact oriented *topological* $2m$-manifold W with boundary M (which may be ϕ). Let $\gamma \in H^1(W; Z)$ and $\dot{\gamma}=\gamma | M \in H^1(M; Z)$. Let $\tilde{W}$ be the infinite cyclic covering space of W associated with γ, whose covering transformation group is infinite cyclic and denoted by $\langle t \rangle$ with a specified generator t (cf. [K3, § 0]). The boundary $\tilde{M}$ of $\tilde{W}$ is the infinite cyclic covering space of M associated with $\dot{\gamma}$ (if it is not ϕ), and we have the signature invariants $\sigma_a^{\dot{\gamma}}(M)$, $a \in [-1, 1]$, of $(M, \dot{\gamma})$ (cf. [K2], [K3]). These signature invariants were defined as a result of a duality on the cohomology ring $H^*(\tilde{M})$. This duality was first observed by Milnor [M], under the restriction that $H^*(\tilde{M})$ is finitely generated over a field. This restriction was removed in [K1]. Neumann [N2] has independently shown it by modifying the Blanchfield linking form. [*Remark*: In [M], [K1] and [N2], it was assumed that M is triangulated, but one can find a proof of its topological version in [K3, Appendix B].] In [K3], the author could compute these signature invariants by using a certain linking matrix on $(M, \dot{\gamma})$. By convention, $\sigma_a^{\dot{\gamma}}(M)=0$ if $M=\phi$. The purpose of this paper is to introduce and compute signature invariants, $\tau_{a\pm 0}^{\gamma}(W)$ of (W, γ), defined for all $a \pm 0$*) $\in [-1, 1]$ (cf. § 1). It turns out that the set $\{\tau_{a\pm 0}^{\gamma}(W) - \text{sign}\, W \,|\, a\pm 0 \in [-1, 1]\}$ and $\{\sigma_a^{\dot{\gamma}}(M) \,|\, a \in [-1, 1],\ a \neq -\varepsilon(m)\}$ determine each other, where $\varepsilon(m)=(-1)^m$ and sign W denotes the usual signature of W (By convention, sign $W=0$ if $\varepsilon(m)=-1$). Moreover, we shall show that $\sigma_{-\varepsilon(m)}^{\dot{\gamma}}(M)$ can be written in terms of $\tau_{-\varepsilon(m)+\varepsilon(m)0}^{\gamma}(W)$, sign W and a certain signature invariant, $\text{sign}_\gamma W$ of (W, γ). Thus, we can see that the signature invariants $\sigma_a^{\dot{\gamma}}(M)$, $a \in [-1, 1]$, are all *peripheral invariants* (the terms due to Neumann [N1]), such as an invariant of Atiyah/Singer [A/S], called α-invariant by Hirzebruch/Zagier [H/Z] and an invariant of Atiyah/Patodi/Singer [A/P/S], called γ-invariant by Neumann [N1], [N2]. The

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*) We use the convention $a+0 \in [-1, 1]$ for $a \in [-1, 1)$ and $a-0 \in [-1, 1]$ for $a \in (-1, 1]$.