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On 3-dimensional Bounded Cohomology of Surfaces

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§ 1. Introduction

In [3], Gromov introduced the notion of the bounded cohomology $H_b^*(M, \boldsymbol{R})$ of a manifold M. This is the cohomology of the complex of singular cochains ϕ which have the property:

There exists a constant c such that $|\phi(\sigma)|<c$ for any singular simplex σ.

Let S be a closed oriented surface of genus $\geqq 2$. In [1] and [5], it is shown that $H_b^2(S, \boldsymbol{R})$ is infinitely generated.

In this paper, we shall show

Theorem 1. *$H_b^3(S, \boldsymbol{R})$ is infinitely generated.*

Our method is an application of Thurston's theory of pleated (uncrumpled) surfaces in hyperbolic 3-manifolds ([7]).

§ 2. A construction of elements of $H_b^3(S, \boldsymbol{R})$

For a convenience, we choose and fix a complete hyperbolic structure on S.

Let f be a pseudo Anosov diffeomorphism of S. Let M_f be the mapping torus of f. It is the identification space obtained from $S\times[0, 1]$ by equivalence relation $(x, 0)\sim(f(x), 1)$ $(x \in S)$. M_f admits a complete hyperbolic structure which is unique up to isometry ([6]). The projection onto the second factor $S\times[0, 1]\rightarrow[0, 1]$ induces a fibering $p: M_f\rightarrow S^1$. Let $\tilde{M}_f$ be the infinite cyclic regular covering space of M_f defined by the pull-back by p of $e: \boldsymbol{R}\rightarrow S^1$, where $e(t)=\exp 2\pi\sqrt{-1}\, t$, $t \in \boldsymbol{R}$. The hyperbolic structure on M_f can be lifted to the hyperbolic structure on $\tilde{M}_f$. There is a natural inclusion $S\times[0, 1)\subset\tilde{M}_f$ and let $j: S\rightarrow\tilde{M}_f$ be the embedding defined by $j(x)=(x, 0) \in S\times[0, 1)\subset\tilde{M}_f$.

Let Δ be the standard 3-simplex in $\boldsymbol{R}^4$. Let $\sigma: \Delta\rightarrow S$ be a singular 3-simplex of S. Then $j\sigma: \Delta\rightarrow\tilde{M}_f$ is a singular 3-simplex of $\tilde{M}_f$. The universal covering space of $\tilde{M}_f$ is isometric to the hyperbolic 3-space H^3,

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