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On Uniformizations of Orbifolds

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§ 1. Introduction

By a *transformation group*, we shall mean a pair (G, M) of a connected paracompact n-manifold M and a group G of topological transformations of M. For a second transformation group (H, N), by a *covering map* (ϕ, f): $(G, M) \to (H, N)$, we shall mean a pair of an epimorphism ϕ: $G \to H$ and a regular covering map f: $M \to N$ which is ϕ-equivariant, i.e., $f(g \cdot z) = \phi(g) \cdot f(z)$ for $(g, z) \in G \times M$. In particular, if f is a homeomorphism, then (ϕ, f) is called an *isomorphism* and (G, M) and (H, N) are said to be *isomorphic*, written $(G, M) \cong (H, N)$. We shall say that (G, M) is a *proper transformation group*, if a track of the action mapping $G \times M \to M \times M$; $(g, z) \mapsto (z, g \cdot z)$ is proper, where G has the discrete topology. In other words, the orbit space $X = \{G \cdot z \mid z \in M\}$ is Hausdorff and (G, M) is discontinuous, i.e., for each point z of M the isotropy subgroup $G_z = \{g \in G \mid g(z) = z\}$ of G at z is finite and there is a neighborhood U_z, referred to as a *G-equivariant neighborhood*, of z in M such that $g \cdot U_z = U_z$ for $g \in G_z$ and $g \cdot U_z \cap U_z = \phi$ for $g \in G - G_z$. A proper transformation group (G, M) is *locally smooth*, if for each point z of M, there is a G-equivariant neighborhood M_z of z in M such that (G_z, M_z) is isomorphic with a finite orthogonal transformation group (G'_z, E_z), i.e., E_z is euclidean n-space $\boldsymbol{R}^n$ or closed half n-space $\boldsymbol{H}_n$ and G'_z is a finite subgroup of $O(n)$.

In this paper, we concern ourselves with *the classification of locally smooth proper transformation groups*. Thus by a transformation group (G, M) we shall mean a locally smooth proper one, unless otherwise mentioned.

Now for a transformation group (G, M), we have the orbit space X, which is a connected separable Hausdorff space, and a function $b: X \to N$

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