Advanced Studies in Pure Mathematics 9, 1986
Homotopy Theory and Related Topics
pp. 129–134

Stiefel-Whitney Homology Classes and Riemann-Roch Formula

Akinori Matsui and Hajime Sato

§ 1. Introduction

In this note, we give a Riemann-Roch type theorem for certain maps between Euler spaces. These are the cases where Halperin's conjecture holds, although it is not true in general [6].

Let X be a locally compact n-dimensional polyhedron. For a point x in X, let $\chi(X, X-x)$ denote the Euler number of the pair $(X, X-x)$. The polyhedron X is called a *mod* 2 *Euler space* or simply an *Euler space* if for each x in X, $\chi(X, X-x)\equiv 1 \pmod 2$ (Halperin and Toledo [3]).

Let K' denote the barycentric subdivision of a triangulation K of a polyhedron X. If X is an Euler space, the sum of all k-simplexes in K' is a mod 2 cycle and defines an element $s_k(X)$ in $H_k(X; Z_2)$ (cf. [3]). The element $s_k(X)$ is called the k-th *Stiefel-Whitney homology class* of X.

In the book [2], Fulton and MacPherson defined the notion of a homologically normally nonsingular map. As an analogy to the Riemann-Roch formula for singular algebraic spaces, they introduced Halperin's conjecture ([2, p. 112]):

If $\phi\colon X\to Y$ is a homologically normally nonsingular map of Euler spaces, then

$$s_*(X)=\phi^! s_*(Y)\cap (wN_\phi)^{-1},$$

where $(wN_\phi)^{-1}$ is the inverse of the cohomology Stiefel-Whitney class of the normal space of ϕ defined by Thom's formula using the Steenrod squares.

If Y is an Euclidean space and ϕ is an embedding, then ϕ is homologically normally nonsingular if and only if X is a Z_2-homology manifold. In this case, Halperin's conjecture is equal to the equation

$$s_*(X)=[X]\cap w^*(X),$$

which is proved by Taylor [8], Veljan [9] and Matsui [4].

Received February 13, 1985.