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Classifying Space Constructions in the Rational Homotopy Theory

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§ 1. Rational homotopy theory

For simplicity's sake, we restrict our descriptions to the 1-connected case. Namely, let TOP_1 be the category of the 1-connected topological spaces, DGL_1 be that of the 1-reduced (*i.e.* $L_i=0$ for $i<1$) differential graded rational Lie algebras, and DGA^1 be that of the 1-connected (*i.e.* $A^i=0$ for $i<0$ and $i=1$, and $A^0\cong \boldsymbol{Q}$) differential graded rational algebras. Throughout these categories the notion of the (rational) weak equivalence is defined as follows.

Definition. (i) A continuous map $f\colon X\to Y$ is *a* (*rational*) *weak isomorphism* if the induced homomorphism on the rational homotopy groups

$$f_\#\colon \pi_*(X)\otimes \boldsymbol{Q}\longrightarrow \pi_*(X)\otimes \boldsymbol{Q}$$

is an isomorphism.

(ii) DGL_1-homomorphism $g\colon (L_*, \partial)\to(L'_*, \partial')$ is *a weak isomorphism* if the induced homomorphism on the homology Lie algebras

$$g_*\colon H_*(L_*, \partial)\longrightarrow H_*(L'_*, \partial')$$

is an isomorphism.

(iii) DGA^1-homomorphism $h\colon (A^*, d)\to(B^*, d')$ is *a weak isomorphism* if the induced homomorphism on the cohomology algebras

$$h^*\colon H^*(A^*, d)\longrightarrow H^*(B^*, d')$$

is an isomorphism.

(iv) In each of the three cases above, the equivalence relation generated by the weak isomorphisms is called *the* (*rational*) *weak equivalence* and the corresponding quotient category of this equivalence relation is denoted by $\mathscr{H}o_{\boldsymbol{Q}}(\_\_)$.

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