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Unique Triangulation of the Orbit Space of a Differentiable Transformation Group and its Applications

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Dedicated to the memory of Shichirô Oka

Introduction

Let G be a compact Lie group throughout this paper. We consider a paracompact differentiable manifold M of class C^k and dimension m with a differentiable G-action $G \times M \to M$ of class C^k, which we call a C^k G-manifold.

We shall see that a differentiable (i.e., C^k with $1 \leqq k \leqq \infty$) G-manifold M is equivariantly diffeomorphic to a real analytic (i.e., C^ω) G-manifold (Theorem 1.3). A C^ω equivariant smoothing is "uniquely" determined (Theorems 1.2–1.2′): unique up to C^ω equivariant diffeomorphism if M is compact or more generally M has only a finite number of orbit types and unique up to subanalytic C^1 equivariant diffeomorphism in general. We use here the equivariant embedding theorem for real analytic G-manifold with finite orbit types in a finite dimensional linear representation space (Theorem 1.1).

Reviewing the notion of subanalytic sets and maps defined by Hironaka [H1] in Section 2, we treat the real analytic G-manifolds in Section 3. A natural subanalytic set structure is introduced on the orbit space (Theorem 3.1) and the stratification filtered by orbit types is subanalytic (Lemma 3.2). So, we have a unique triangulation of the orbit space which is compatible with the subanalytic set structure and consequently with the orbit type decomposition in the sense that two such triangulations have a common subanalytic and combinatorial subdivision (Theorem 3.3) using the results of [SY].

Combining these results we get a unique triangulation of the orbit space also for any differentiable G-manifold M. Notice that the orbit space of a differentiable G-manifold M with boundary is nothing but that of the differentiable $G \times Z_2$-manifold DM, where DM is the double of M;

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