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Stable Equivalence of G-Manifolds

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Dedicated to Professor Minoru Nakaoka on his 60th birthday

§ 1. Introduction

Let G be a compact Lie group and M_1, M_2 be closed G-manifolds. Suppose that there exists a G-homotopy equivalence

$$f\colon M_1 \longrightarrow M_2.$$

Then a natural question is the following. What kind of consequences follow from it?

For example, the following theorem holds.

Theorem 1 [14], [16]. *We have the following equality in* $J_G(M_1)$:

$$J_G(T(M_1)) = J_G(f^*(T(M_2))).$$

Here $T(M_i)$ *denote the tangent G-vector bundles of* M_i $(i=1, 2)$ *and*

$$J_G\colon KO_G(M_1) \longrightarrow J_G(M_1)$$

denotes the equivariant J_G*-homomorphism.*

It is well-known that G-homotopy equivalent manifolds are not necessarily G-diffeomorphic in general [5], [13], [15].

Our first result of the present paper is the following theorem.

Theorem 2. *Let* M_1 *and* M_2 *be closed G-manifolds. If* $f\colon M_1 \to M_2$ *is a G-homotopy equivalence, then there exist G-vector bundles* $\pi_i\colon E_i \to M_i$ $(i=1, 2)$ *and a G-diffeomorphism*

$$\bar{f}\colon E_1 \longrightarrow E_2$$

such that the following diagram

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