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Equivariant Whitehead Groups and G-Expansion Categories

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Dedicated to Professor Minoru Nakaoka on his 60th birthday

Introduction

As to the classical theory of Whitehead torsions and Whitehead groups the original works by J.H.C. Whitehead [21], [22], [23] and the expository work by J. Milnor [12], 1966, is celebrated. Since then there appeared about 1970 the trials [5], [17] to define Whitehead group $\mathrm{Wh}(X)$ of a space X and to prove the isomorphism

$$\mathrm{Wh}(X) \cong \mathrm{Wh}(\pi_1 X),$$

where the right hand side is the classical algebraically defined Whitehead group of the fundamental group of X.

In 1974 S. Illman [7] defined equivariant Whitehead group of a finite G-CW-complex X, denoted by $\mathrm{Wh}_G(X)$, where G is a compact Lie group, equivariant Whitehead torsion for a G-homotopy equivalence $f: X \to Y$ between finite G-CW-complexes X, Y as an element of $\mathrm{Wh}_G(X)$, and described the basic properties of $\mathrm{Wh}_G(X)$. He tried also to decompose $\mathrm{Wh}_G(X)$ and to describe it algebraically for abelian G and proposed to use restricted Whitehead groups $\mathrm{Wh}_G(X, \mathscr{F})$ with respect to families $\mathscr{F}$ of closed subgroups of G in studies of equivariant Whitehead group of X.

In 1978 M. Rothenberg [13] defined equivariant Whitehead groups and torsions in another way for finite G and obtained several results making use of them. Among others he obtained a form of equivariant s-cobordism theorem for smooth actions of compact Lie groups G in 3.4 by introducing certain other invariants $\mathrm{Wh}_H^i(\iota)$ for closed subgroups H of G. In case G is finite and $X^{(H)}$ is connected and simply connected for all subgroups H of G he proved the collection of the invariants $\mathrm{Wh}_H(\iota)$ coincides with his G-Whitehead torsion in 3.10. Here we remark that our definition of G-h-cobordism [3], [9] is different from that of Rothenberg's. Probably, under the assumption of our dimension gap

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