Advanced Studies in Pure Mathematics 8, 1986
Complex Analytic Singularities
pp. 671–685

On Plurigenera of Normal Isolated Singularities II

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In this paper we prove some results on plurigenera of normal isolated singularities. This paper is a continuation of [19].

In Section 1, we recall some preliminary facts related to the concept of plurigenera of normal isolated singularities. In Section 2, we prove the δ_m-formula for non-degenerate hypersurface isolated singularities, which is a generalization of Theorem 1.13 [19, p. 71]. In Section 3, we determine the "type" of purely elliptic singularities of hypersurfaces. In the last section we show a criterion for a singularity to be Du Bois: In the case where a singularity is quasi-Gorenstein, a singularity (X, x) is Du Bois if and only if $0 \leqq \delta_m(X, x) \leqq 1$. Finally we give examples of Du Bois singularities with possibly positive geometric genera, which are not quasi-Gorenstein.

§ 1. Plurigenera of normal isolated singularities

We need to recall a few preliminaries related to the concept of plurigenera of normal isolated singularities. For more details we refer to [19].

Let (X, x) be a normal isolated singularity of an n-dimensional analytic space X. Let V be a (sufficiently small) Stein neighborhood of x and let K be the canonical line bundle of $V-\{x\}$. For convenience, we denote the line bundle $K^{\otimes m}$ by mK. An element of $\Gamma(V-\{x\}, \mathcal{O}(mK))$ is considered as a holomorphic m-ple n-form. Let ω be a holomorphic m-ple n-form on $V-\{x\}$. We write ω as

$$\omega = \phi(z)(dz_1 \wedge dz_2 \wedge \cdots \wedge dz_n)^m,$$

using local coordinates $(z_1, z_2, \cdots, z_n)$. We associate with ω the continuous (n, n)-form $(\omega \wedge \bar{\omega})^{1/m}$ given by

$$|\phi(z)|^{2/m}(\sqrt{-1}/2) dz_1 \wedge d\bar{z}_1 \wedge dz_2 \wedge d\bar{z}_2 \wedge \cdots \wedge dz_n \wedge d\bar{z}.$$

Received April 1, 1985.
Revised November 1, 1985.