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Three-Dimensional Cusp Singularities

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Introduction

2-dimensional cusp singularities are characterized by the exceptional sets of their resolutions. Namely, a 2-dimensional isolated singularity (V, p) is a cusp singularity, if and only if it has a resolution $(U, X) \to (V, p)$ whose exceptional set X is a cycle of rational curves or a rational curve with a node. (See, for instance, Laufer [2].) Thus assigning to each 2-dimensional cusp singularity the weighted dual graph of the exceptional set X of its unique minimal resolution, we have a one-to-one correspondence between the set of all isomorphism classes of 2-dimensional cusp singularities and the set of all weighted triangulations of circles whose weights are not greater than -2 and at least one of whose weights is strictly smaller than -2. Here the weighted dual graph of X is the dual graph of X, in the usual sense, to each vertex of which is attached the self-intersection number of the corresponding rational curve as the weight, when X is a cycle of rational curves. When X is a rational curve with a node, the weighted dual graph of X is a circle with one vertex, to which is attached X^2-2 as the weight.

The purpose of this paper is to find out the 3-dimensional analogues.

First, we define toric divisors as analogues of the exceptional sets of resolutions of 2-dimensional cusp singularities (Definition 1.1). For example, all the isolated cusp singularities which appear in compactifications of quotient spaces of tube domains have resolutions whose exceptional sets are toric divisors (see [1]). Then we define a "cusp" singularity to be a 3-dimensional isolated singularity having a resolution whose exceptional set is a toric divisor. Next, we define the weighted dual graph of a toric divisor, which is a graph on a compact topological surface with two integers attached to each edge as weights (Definition 1.2). Then we have the following result (Theorem 1.6), which we prove in Section 2:

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