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Maximal-Ideal-Adic Filtration on $R^1\psi_*O_{\tilde{V}}$ for Normal Two-Dimensional Singularities

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§ 0. Introduction

(0.1) Let (V, p) be a germ of normal two-dimensional algebraic variety over $\boldsymbol{C}$ at a reference point p. We simply call it a normal two-dimensional singularity. Let $\psi: (\tilde{V}, A) \to (V, p)$ be a resolution of the singularity (V, p) with exceptional set A. It is well-known that the coherent O_V-module $R^1\psi_*O_{\tilde{V}}$ is independent of the choice of resolution. The *geometric genus* of the singularity (V, p) is the integer $p_g(V, p)$ defined by: $p_g(V, p) = \dim R^1\psi_*O_{\tilde{V}}$. This number has been studied by many authors from many viewpoints (cf. [9, 10, 11, 12, 15, 17, 19, 20, 21, 22, 23, 24] and the references there).

In this paper, we shall study the O_V-module $R^1\psi_*O_{\tilde{V}}$ itself. More precisely, we shall study the numerical invariants which are related to the following maximal-ideal-adic filtration on $R^1\psi_*O_{\tilde{V}}$:

$$(*) \qquad R^1\psi_*O_{\tilde{V}} \supseteqq m \cdot R^1\psi_*O_{\tilde{V}} \supseteqq \cdots \supseteqq m^L \cdot R^1\psi_*O_{\tilde{V}} = 0,$$

where m denotes the maximal ideal of $O_{V,p}$. We define the invariant $L(V, p)$ as the length of the filtration above. Since $m^i \cdot R^1\psi_*O_{\tilde{V}} \neq m^{i+1} \cdot R^1\psi_*O_{\tilde{V}}$ for non-zero $m^i \cdot R^1\psi_*O_{\tilde{V}}$, this integer can be written as follows (see also (0.2) and (2.9)): $L(V, p) = \min\{r \in \boldsymbol{Z} \mid r \geqq 0,\ m^r \cdot R^1\psi_*O_{\tilde{V}} = 0\}$.

First we shall show the existence of an element f of m such that the equalities $m^r \cdot R^1\psi_*O_{\tilde{V}} = f^r \cdot R^1\psi_*O_{\tilde{V}}$ for $r \geqq 0$ hold. Hence the filtration (*) is determined by the nilpotent endomorphism

$$F: R^1\psi_*O_{\tilde{V}} \longrightarrow R^1\psi_*O_{\tilde{V}};\ \alpha \longmapsto f \cdot \alpha \text{ (Section 1 and (2.3)).}$$

At the same time, by using the divisor $D(m, \psi)$ which is called *maximal ideal cycle* in [24, 17, 19] we can show the equality $(R^1\psi_*O_{\tilde{V}}/m \cdot R^1\psi_*O_{\tilde{V}}) \cong H^1(O_{D(m,\psi)})$. In particular, we obtain the equalities $\dim(R^1\psi_*O_{\tilde{V}}/$

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