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# The Versality Theorem for *RL*-Morphisms of Foliation Unfoldings

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In [7], we proved a versality theorem for unfoldings of codim 1 foliation germs, which generalizes the versality theorem with respect to right morphisms in the unfolding theory of function germs. The purpose of this paper is to prove a similar theorem with respect to *RL*-morphisms. These morphisms generalize right-left morphisms in the function case and play an important role in the determinacy problem of foliation germs (see [11]). We also note that the definition naturally involves integrating factors of the given foliation germ (see Definitions (1.1) and (1.2) and (1.3) Remark).

In Section 1, we recall terminologies and describe the set of *RL*-isomorphism classes of first order unfoldings of a foliation germ. We prove, in Section 2, the versality theorem ((2.1) Theorem), which says that an infinitesimally *RL*-versal unfolding of a codim 1 foliation germ $F$ is *RL*-versal. Let $\mathscr{F}$ be an infinitesimally *RL*-versal unfolding of $F$ and let $\mathscr{F}'$ be an arbitrarily given unfolding of $F$. The proof consists of, as in [7], (I) construction of an *RL*-morphism from $\mathscr{F}'$ to $\mathscr{F}$ as a formal power series in the parameters of $\mathscr{F}'$ and (II) proof of the existence of a convergent solution. Basically the infinitesimal *RL*-versality is sufficient for (I), although the procedure is rather involved. For (II), we need some side condition (( * ) in (2.1) Theorem, see also (2.2) Remark), which is satisfied in most cases. We compare the series obtained in (I) with convergent series similar to the one used in Kodaira-Spencer [6]. For this, we use the privileged neighborhood theorem of Malgrange [3] as well as the unfolding theory of integrating factors developed in the appendix of this paper. In Section 3, we explain how our theorem is related to the versality theorem for unfoldings of function germs with respect to right-left morphisms (cf. Wassermann [13]). We consider the "meromorphic" case in Section 4. Namely, for a foliation $F$ generated by a germ $\omega$ of the form $\omega = gdf - fdg$, where $f$ and $g$ are holomorphic function germs, we determine the set of integrating factors ((4.1) Lemma) and apply (2.1) Theorem to obtain an *RL*-universal unfolding of $F$ explicitly ((4.6)

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