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Sandwiched Surface Singularities And the Nash Resolution Problem

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§ 1. Introduction

Let X be an algebraic variety over $\boldsymbol{C}$. Consider the tower of morphisms

$$(*) \qquad \cdots \longrightarrow X^{(i+1)} \xrightarrow{\nu_i} X^{(i)} \longrightarrow \cdots \longrightarrow X'' \xrightarrow{\nu_1} X' \xrightarrow{\nu} X$$

where either all the ν_i are Nash modifications (abbreviated by N) or Nash modifications followed by normalizations (abbreviated by NN).

The Nash problem: Is $X^{(i)}$ nonsingular for $i \gg 0$?

It is known ([7], p. 300) that, in characteristic 0, N is an isomorphism if and only if X is nonsingular. In particular, if $\dim X=1$, a sequence of N desingularizes.

In this paper, we discuss the following.

Theorem 1.1. *Let* $\dim X=2$. *Then a sequence of NN desingularizes.*

For the rest of this paper all the varieties will be 2-dimensional algebraic varieties over $\boldsymbol{C}$ unless otherwise specified.

All the ν_i's in $(*)$ will be NN.

The following partial results were known previously:

Theorem 1.2 (González-Sprinberg, [2] pp. 176, 129–136). *If the singularities of X are rational double points or cyclic quotients, a sequence of NN desingularizes.*

Theorem 1.3 (Hironaka, [3], p. 110). *For any surface X, consider the sequence* $(*)$. *Then, for $i \gg 0$, $X^{(i)}$ birationally dominates a nonsingular surface* (*namely, the minimal resolution of X*).

Theorem 1.3 motivates the following definition:

Definition 1.1. *Let* $(\mathcal{O}, \mathcal{M})$ *be a normal local ring. We say that $\mathcal{O}$ has*

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