A Stratification Theoretical Method of Construction
of Holomorphic Vector Bundles

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In this paper, we propose an explicit method of construction of holomorphic vector bundles over a complex variety. In the construction, our guiding model is the universal quotient bundle over Grassmann variety. The content of this paper is rather provisional and experimental, but may be used as a general method for treatments of bundles.

Introduction

1. Letting $\bar{X}$ be a normal complex variety, the purpose of this paper is to construct holomorphic vector bundles over $\bar{X}$, by the following two steps:

(I) To find a bundle E_X over $X = \bar{X} - \{$a codimension two subvariety of $\bar{X}\}$, which is endowed with a suitable '*stratification theoretical representation*', and

(II) to investigate structure of the direct image $E_{\bar{X}} = i_*E_X$, i being the injection: $X \hookrightarrow \bar{X}$, by giving a similar representation to the one in (I). (See Section 0 for more details of (I) and (II).)

Encouraging facts for our proposed approach are: (i) If $\bar{X}$ is a quasi projective or a Stein variety, then each bundle over $\bar{X}$ is obtained in the manner (I), (II), and (ii) the procedure (I), (II) may be regarded as a generalization of classical methods in treatments of bundles over a Riemann surface [Bir], [Weil] and [Tj] (cf. § 0).

2. The content of this paper is briefly as follows: In Section 1 we give some explicit coherent sheaf theoretical expressions of the bundle E_X as in (I). In Section 2, we introduce the notion of '*type* (G)' for such a bundle and give some basic properties of the bundle. A bundle E_X of type (G) is, in our context, an abstraction of a *bundle obtained as the pull back of the universal quotient bundle over a Grassmann variety*. The main results of this paper are given to such a bundle E_X and are as follows: