Advanced Studies in Pure Mathematics 8, 1986
Complex Analytic Singularities
pp. 405–436

On the Resolution of the Hypersurface Singularities

Mutsuo Oka

Dedicated to Professor M. Nakaoka on his 60th birthday

§ 1. Introduction

Let $f(z_0, \cdots, z_n)$ be a germ of an analytic function at the origin such that $f(0)=0$ and f has an isolated critical point at the origin. We assume that the Newton boundary of f is non-degenerate. Let V be the germ of the hypersurface $f^{-1}(0)$ at the origin. Let $\Gamma^*(f)$ be the dual Newton diagram and let Σ^* be a simplicial subdivision. It is well-known that there is a canonical resolution $\pi: \tilde{V} \to V$ which is associated with Σ^* ([8]). However the process to get Σ^* from $\Gamma^*(f)$ is not unique and a "bad" Σ^* produces unnecessary exceptional divisors. The purpose of this paper is to study this resolution through a canonical simplicial subdivision.

In Section 3, we will show that there is a canonical way to get a simplicial subdivision from $\Gamma^*(f)$. (Lemma (3.3) and Lemma (3.8))

In Section 4, we will recall the construction of the resolution $\pi: \tilde{V} \to V$ which is associated with a given simplicial subdivision Σ^*.

In Section 5, we will study the topology of the exceptional divisors using the canonical stratifications.

In Section 6, we will show the following: Assume that $n=2$. Then the resolution graph Γ of the resolution of V is obtained by a canonical surgery from $S_2\Gamma^*(f)$ (=the two-skeleton of $\Gamma^*(f)$ which is considered as a graph by a plane section). Let P be a vertex of Σ^* such that $\Delta(P)$ is a two-dimensional face of $\Gamma(f)$. Then the genus of the exceptional divisor $E(P)$ is equal to the number of the integral points in the interior of $\Delta(P)$. The other exceptional divisors are rational. (See Theorem (6.1) of § 6.)

In Section 7, we will study the fundamental group of the exceptional divisor $E(P)$. Assume that $n>2$ and $\Delta(P)$ is an n-simplex. Then we will show that $\pi_1(E(P))$ is a finite cyclic group and its order is determined by $\Gamma^*(f)$ (Theorem (7.3)).

In Section 8, we will study the divisors of the exceptional divisor $E(P)$ in the case of $n=3$.

Received December 26, 1984.