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Infinitely Very Near Singular Points

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To the memory of Professor Yasuo Akizuki

Introduction

The purpose of this paper is to explain the current state of Hironaka's program for the characteristic-free resolution of singularities of excellent schemes. More specifically, we collect together results on infinitely near points and infinitely very near points due to Bennett [B]. Giraud [G_1], [G_2]. Hironaka [H_1], [H_4]. Herrmann-Orbanz [HO], Oda [O_4] and Singh [S_1], [S_2], [S_3]. We remove unnecessary restrictions and give unified proofs for most of them, using the ideas found in the above papers. Hopefully, we thus get insight into possible further study of the program.

Hironaka's program consists in

(I) finding good numerical invariants for singularities which always improve or at least remain the same under any permissible blowing up, and

(II) finding a finite succession of permissible blowing ups which actually improves these numerical invariants at singular points of a given excellent scheme.

Hironaka successfully carried out this program for excellent surfaces (see [H_2] for a sketch). It might be possible to carry out (II) either

(II_1) in a mesh of inductions on the dimension as in Hironaka's proof [H_1] in characteristic zero, or

(II_2) in formulating a good game on Newton polyhedra and then finding a winning strategy for it.

Spivakovsky dealt with a prototype of this game formulated by Hironaka and in [S_5] found a winning strategy for a simpler version, while in [S_4] he showed that a winning strategy need not exist for a harder version. Nothing else seems to be known about (II) at the moment.

Our main concern in this paper is (I) for general excellent schemes. We have already sketched in [O_3], without proof, the current state of (I)

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