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Splicing Algebraic Links

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§ 1. Introduction

In this paper we give an introduction to the terminology of splicing (see "Three-dimensional link theory and invariants of plane curve singularities" by Eisenbud and Neumann, [EN]) and then describe how to compute a normal form representation of the real monodromy and Seifert form for the link of a plane curve singularity from this point of view (it was done via a resolution diagram for the singularity in [N3]). It has been conjectured that this might be a complete invariant for the topology of an isolated complex hypersurface singularity in any dimension; the originator now denies responsibility and will remain unnamed, but the conjecture is still unresolved. Many of the required invariants are computed in [EN] and we just review these computations. The first four sections and Theorem 5.1 are survey and review; the main new result is the computation of the equivariant signatures of the monodromy via splicing in Theorem 5.3. This computation applies also to general graph links.

A *link* for us is a pair (Σ, K) where Σ is an oriented homology 3-sphere and K is a disjoint union of oriented circles in Σ. Let (V, p) be a germ of a normal complex surface at a $\boldsymbol{Z}$-homology manifold point, that is $H^*(V, V-p; \boldsymbol{Z}) = H^*(\boldsymbol{C}^2, \boldsymbol{C}^2-0; \boldsymbol{Z})$. Let $f\colon (V, p) \to (\boldsymbol{C}, 0)$ be the germ of an analytic map. We may assume (V, p) embedded in some ambient $(\boldsymbol{C}^n, 0)$ and then by intersecting $(V, f^{-1}(0))$ with a sufficiently small sphere about $0 \in \boldsymbol{C}^n$, we obtain the *link* $(\Sigma, K(f))$ of f. We call such a link an *algebraic graph link*; if $(V, p) = (\boldsymbol{C}^2, 0)$, it is just the link of a plane curve singularity. We make no reducedness assumption on f; thus each branch of $f^{-1}(0)$, and correspondingly each component of $K(f)$, carries a positive integer multiplicity; in the terminology of [EN], $(\Sigma, K(f))$ is a *multilink*. A link is the special case of a multilink with all multiplicities equal to 1.

The invariants we are interested in are invariants of the Milnor

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