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An Upper Semicontinuity Theorem for some Leading Poles of $|f|^{2s}$

Ben Lichtin

Introduction

In this paper an application is made of certain numerical invariants introduced by Libgober [9], called "quasi-adjoint characters". To each germ of an analytic function f at a singular point p and to any other germ of an analytic function ϕ at p one may define the quasi-adjoint character $\kappa_\phi(p)$ by studying the family of cyclic covers over f and the adjointness properties to these cyclic covers of canonical differentials with ϕ as a coefficient (for precise definitions see (2. 4)). Each $\kappa_\phi(p)$ value is in $[0, 1)$.

The main result of this paper is the

Theorem (3.1). *Let $\{f_t\}$ be any 1-parameter family of germs of analytic functions at the common singular point $\bar{0} \in C^n$. Let ϕ be a germ of an analytic function at $\bar{0}$. Let $\kappa_\phi(t)$ be the quasi-adjoint character associated to f_t and ϕ at $\bar{0}$. Then, if $\kappa_\phi(0) \in (0, 1)$, one has*

$$\kappa_\phi(t) \leq \kappa_\phi(0)$$

for all t sufficiently close to 0.

This is of particular interest because of the following. For each t, let $U_t \Subset U'_t$ be two Milnor balls for a representative of f_t (denoted by f_t). Let ρ be a C^∞ function which is 1 on U_t and 0 off U'_t. Define the generalized functions on $C^\infty(U'_t, C)$ $I_t(s, \psi) = \int_{U'_t} |f_t|^{2s} |\psi|^2 \rho dx d\bar{x}$. This is often denoted by $|f_t|^{2s}$ for short. Let $\beta_\phi(t)$ be the largest pole of $I_t(s, \phi)$. Then there is a simple relation between $\kappa_\phi(t)$ and $\beta_\phi(t)$ given by $\kappa_\phi(t) + 1 = \beta_\phi(t)$ if $\kappa_\phi(t) \in (0, 1)$. Thus, (3.1) implies as a corollary

Corollary (3.8). *If $\kappa_\phi(0) \in (0, 1)$ then $\beta_\phi(t) \leq \beta_\phi(0)$ for t near* 0.

To understand this condition it is helpful to remark that if ϕ is a local unit at $\bar{0}$, then $\kappa_\phi = 0$ iff $\bar{0}$ is a rational singular point of f. More generally, $\kappa_\phi = 0$ iff ϕ is adjoint to f at $\bar{0}$.

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