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On π_2 of the Complements to Hypersurfaces which are Generic Projections

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§ 1. Introduction

Let V denote a non-singular projective variety of dimension n embedded into a projective space of dimension N. Let L be a generic linear subspace of CP^N of dimension $N-n-2$. The projection with center at L defines a map $p: V \to CP^{n+1}$. The purpose of this note is to prove the following:

Main Theorem. *If V is a simply-connected non-singular variety of dimension greater than one then $\pi_2(CP^{n+1}-p(V))\otimes Q$ is trivial.*

Several comments are in order to explain our interest in such kind of a result. The variety $p(V)$ is a singular hypersurface having singularities which are fairly well understood (at least in the case of small dimensions) [R1]. For example if $n=2$ then $p(V)$ has singularities along certain curve (double curve) near which $p(V)$ given by $xy=0$. Moreover it has finitely many triple points given locally by equation $xyz=0$ and finite number of pinch points locally given by $x^2-yz^2=0$. In particular this implies that if H is a generic plane in CP^{n+1} then $H\cap p(V)$ is an irreducible plane curve which has as singularities only nodes. Therefore $\pi_1(H-H\cap p(V))$ is isomorphic to Z/dZ where d is the degree of V ([D]). According to Zariski's theorem [Z], this implies that $\pi_1(CP^{n+1}-p(V))=Z/dZ$. Similarly if H is a generic linear 3-space, then $\pi_2(H-H\cap p(V))$ is isomorphic to $\pi_2(CP^{n+1}-p(V))$ and hence it is enough to prove our theorem only in the case $n=2$.

For a surface in CP^3 with isolated singularities the second homotopy group of the complement has properties similar to the properties of the Alexander modules attached to the fundamental groups of the complements to plane algebraic curves ([L1], [L2]). If S is a non-singular surface in CP^3 then $\pi_2(CP^3-S)$ is trivial, but in general $\pi_2(CP^3-S)$ is affected by the degree of the surface, by the type and by the position in CP^3 of the

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