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Strong Simultaneous Resolution for Surface Singularities

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Let $\lambda: \mathscr{V} \to T$ be (the germ of) a (flat) deformation of the two-dimensional isolated hypersurface singularity (V, p). We take T to be reduced. In [9], Teissier introduced, for all dimensions, various notions of simultaneous resolution for λ. Namely, let V_t denote $\lambda^{-1}(t)$, the fiber above t in T.

Definition 1. The map germ $\Pi: \mathscr{M} \to \mathscr{V}$ is *very weak* simultaneous resolution of λ if for all sufficiently small representatives of λ, the germ Π has a representative, also denoted Π, such that

(0) Π is a proper modification map.

(i) $\lambda \circ \Pi: \mathscr{M} \to T$ is a flat map.

(ii) $\Pi_t: M_t \to V_t$ is a resolution of V_t for all t.

Take V to have dimension two.

Let $\mathscr{A}$ denote the exceptional set in $\mathscr{M}$.

(W) Π is a *weak* simultaneous resolution if additionally the map induced by restriction $\widetilde{\lambda \circ \Pi}: \mathscr{A} \to T$ is simple, i.e. a locally trivial deformation.

Let $\mathscr{S}$ denote the singular locus of $\mathscr{V}$. Consider $\Pi^{-1}(\mathscr{S})$ as a non-reduced analytic space (with $\mathscr{A}$ as its underlying reduced space).

(S) Π is a *strong* simultaneous resolution if in addition to (0), (i) and (ii), the map induced by restriction $\widetilde{\lambda \circ \Pi}: \Pi^{-1}(\mathscr{S}) \to T$ is simple.

(F) Π is a *flat* simultaneous resolution if in addition to (0), (i), and (ii), the map induced by restriction $\widetilde{\lambda \circ \Pi}: \Pi^{-1}(\mathscr{S}) \to T$ is flat.

In [4] (see also [7]), very weak simultaneous resolution (after base change) and weak simultaneous resolution were each shown to be equivalent to the constancy as a function of t of suitable numerical invariants of the fibers. In this paper, it is shown, Theorem 1, that $\mu^*(V_t)$ constant implies strong simultaneous resolution for λ. It is known, [9], in all dimensions, that strong simultaneous resolution implies the Whitney conditions and, [8] [2], that the Whitney conditions are equivalent to $\mu^*(V_t)$ constant. So we complete an affirmative answer in dimension two to

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