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Regular Holonomic D-modules and Distributions on Complex Manifolds

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§ 0. Introduction

Let $(X, \mathcal{O}_X)$ be a complex manifold and $\mathcal{D}_X$ the sheaf of differential operators on X. The de Rham functor $\mathcal{DR}_X = \boldsymbol{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{O}_X, *)$ gives an equivalence of the category $\mathbf{RH}(\mathcal{D}_X)$ of regular holonomic $\mathcal{D}_X$-modules and the category $\mathbf{Perv}(\boldsymbol{C}_X)$ of perverse sheaves of $\boldsymbol{C}$-vector spaces on X ([K], [M], [B-B-D]).

To a perverse sheaf $F^{\cdot}$ on X we can associate its complex conjugate $\bar{F}^{\cdot}$. Then it is easily checked that $\bar{F}^{\cdot}$ is also perverse. We shall discuss here how to construct the corresponding functor c: $\mathbf{RH}(\mathcal{D}_X) \to \mathbf{RH}(\mathcal{D}_X)$ given by $\overline{\mathcal{DR}_X(\mathcal{M})} = \mathcal{DR}_X(\mathcal{M}^c)$.

The solution to this problem is given as follows. Let $\bar{X}$ be the complex conjugate of X and $\bar{\mathcal{M}}$ the complex conjugate of $\mathcal{M}$ (See § 1). Denoting by $\mathcal{D}b_{X_{\boldsymbol{R}}}$ the sheaf of distribution on the underlying real manifold $X_{\boldsymbol{R}}$ of X, $\mathcal{M}^c$ is given by

$$\mathcal{T}or_n^{\mathcal{D}_{\bar{X}}}(\Omega_{\bar{X}}^n \underset{\mathcal{O}_{\bar{X}}}{\otimes} \mathcal{D}b_{X_{\boldsymbol{R}}}, \bar{\mathcal{M}})$$

where $n = \dim X$ and $\Omega_{\bar{X}}^n$ denotes the sheaf of the highest degree differential forms on $\bar{X}$.

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§ 1. The complex conjugate

Let $\bar{X}$ be the complex conjugate of a complex manifold X. Hence $(\bar{X}, \mathcal{O}_{\bar{X}})$ is isomorphic to $(X, \mathcal{O}_X)$ as an $\boldsymbol{R}$-ringed space but the isomorphism $-: \mathcal{O}_X \to \mathcal{O}_{\bar{X}}$ is $\boldsymbol{C}$-anti-linear, i.e. $\overline{af} = \bar{a}\bar{f}$ for $a \in \boldsymbol{C}$ and $f \in \mathcal{O}_X$.

Let $\mathcal{D}_X$ and $\mathcal{D}_{\bar{X}}$ denote the sheaves of differential operators on X and $\bar{X}$, respectively. Then they are isomorphic as a sheaf of $\boldsymbol{R}$-rings. This isomorphism is also denoted by $-$. Through this isomorphism, we can associate the $\mathcal{D}_{\bar{X}}$ module $\bar{\mathcal{M}}$ to a $\mathcal{D}_X$-module $\mathcal{M}$. We call it the complex conjugate of $\mathcal{M}$. The $\mathcal{D}_{\bar{X}}$-module $\bar{\mathcal{M}}$ is isomorphic to $\mathcal{M}$ as a

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