Advanced Studies in Pure Mathematics 8, 1986
Complex Analytic Singularities
pp. 153–163

Du Bois Singularities on a Normal Surface

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Introduction

The purpose of this paper is to investigate Du Bois singularities on normal analytic surfaces. Du Bois introduced the concept of what we call Du Bois singularity by means of his differential complex ([2]). A normal isolated singularity (X, x) is Du Bois if and only if the canonical maps $R^i f_* \mathcal{O}_{\tilde{X}} \to H^i(E, \mathcal{O}_E)$ are bijective for all $i > 0$, where $f: \tilde{X} \to X$ is a good resolution meaning that the divisor $E = f^{-1}(x)_{\mathrm{red}}$ is of normal crossings.

In this paper, a Du Bois singularity is characterized by the property that any holomorphic 2-form on $\tilde{X} - E$ has poles on E of order at most one (Theorem 1.8). Then, we show that any resolution of a Du Bois singularity is a good resolution. We also show that any connected subdivisor of the fibre $E = f^{-1}(x)_{\mathrm{red}}$ of a Du Bois singularity (X, x) is contracted to a Du Bois singularity.

In Theorem 3.2, we get a numerical sufficient condition for a connected configuration of curves with normal crossings on a non-singular surface to be contracted to a Du Bois singularity, which gives examples of Du Bois singularities with arbitrary geometric genus. However, Du Bois' condition is not completely determined by numerical conditions on E. In Section 4, we have examples of Du Bois and non-Du Bois singularities with the same numerical conditions on E (Proposition 4.2).

In this paper, we work only on surface singularities, so, "a singularity" always means a normal singularity on an analytic surface.

The author would like to thank K. Watanabe for providing many examples which helped her to prepare this paper and would also like to express her gratitude to the referee for constructive suggestions.

§ 1. General 2-forms around a Du Bois singularity

In this section, we introduce the concept of a general 2-form around

Received January 4, 1985.
Revised June 18, 1985.
*) Partially supported by JSPS fellowship.