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# On the Singular Subspace of a Complex-Analytic Variety

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## § 1. Introduction

Theme of this paper is the following question:

"How does a germ of a complex-analytic variety relate to its singular subspace?"

The main point herein is to consider the singular locus of a variety not just as a set but provided with a suitable analytic structure. It thus becomes an in general non reduced subvariety of the initial variety, the singular subspace. There are several choices of analytic structures. We shall specify them in a moment. For each of those one can look at the information it contains about the variety one started with. It turns out that certain of them offer a precise insight into the local geometry of the variety around the singularity. We shall describe a number of related results and formulate some open questions.

The presentation avoids technical details and concentrates on the intuitive approach to the problem. Thus the article should be rather understood as a conceptual overview on the situation than a detailed discussion. For the latter, we refer the reader to [G-H].

## § 2. The singular subspace of a hypersurface singularity

Let $(X, 0)$ denote the germ of a hypersurface in $(\boldsymbol{C}^n, 0)$ defined by the analytic equation $f(x)=0$. If $(X, 0)$ is reduced, the set $L$ of singular points of $(X, 0)$ is given by the vanishing of all first order partial derivatives $\partial_1 f, \cdots, \partial_n f$ of $f$. Denote by $j(f)=(\partial_1 f, \cdots, \partial_n f)\subset\mathcal{O}_n$ the jacobian ideal of $f$. There are essentially four different ways to make $L$ into an analytic variety:

( 1 ) $L(X, 0)$ the reduced analytic variety of local ring $\mathcal{O}_{L(X,0)}=\mathcal{O}_n/\sqrt{j(f)}$.

( 2 ) $M(X, 0)$ the variety whose local ring is the Milnor algebra $\mathcal{O}_{M(X,0)}=\mathcal{O}_n/j(f)$.

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