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# A Conjecture about Compact Quotients by Tori

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Let $T \times X \rightarrow X$ be a meromorphic action of $T = (\boldsymbol{C}^*)^k$ on a reduced compact normal complex analytic space $X$. The following is a basic problem of geometric invariant theory whose answer is not known even if the above is an algebraic action on a projective variety $X$.

**Problem.** *Let $T \times X \rightarrow X$ be as above. Classify all $T$ invariant open sets $\mathscr{U} \subseteq X$ such that geometric quotient*

$$\mathscr{U} \longrightarrow \mathscr{U}/T$$

*exists as a compact complex analytic space.*

In this paper we define a finite regular $k$ complex, $\mathscr{C}(X)$, that we call **the moment complex.** We give a conjectural answer to the above question in terms of this $k$ complex and a proof of part of the conjecture. We also discuss the classification of semi-geometric quotients in terms of this complex. For simplicity we assume that $X$ is irreducible and that $X$ can be equivariantly embedded into a complex Kähler manifold on which $T$ acts. This latter assumption, which is always true if $X$ is a normal projective variety, allows us to use moment functions to simplify arguments (cf. § 0).

The previous work on this problem consists of 3 parts:

a) as a special case of his geometric invariant theory [11], Mumford gave a prescription for construction of some of those quotients with $\mathscr{U}/T$ projective,

b) for $k=1$ a complete and simple answer is given in [2] for very general $X$ [see also 4, 6],

c) for $k=2$ a complete answer has recently been given [see 3] in the case that $X$ is a compact Kähler manifold.

The answers in b) and c) are given in terms of special cases of the moment complex, $\mathscr{C}(X)$.

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