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Complex Analytic Singularities
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Characteristic Pairs

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Let us consider an analytic curve in the local complex plane. Let us suppose that our curve is analytically irreducible at the origin and has a singularity there.

In my paper on Quasi-rational Singularities in the *American Journal,* I had noted that if our curve has only one characteristic pair then its singularity can be almost resolved by a covering, i.e., we can find a ramified covering of our local plane by another local plane so that the inverse image of our curve on the covering plane has an ordinary multiple point; in other words, all of its analytic components are regular and their tangent lines are distinct. In that paper I had also asked whether the converse of this is true. In a recent article in the *American Journal,* Libgober has given a topological proof of this converse.

Now I have obtained an algebraic proof of the said converse, i.e., of the fact that if the singularity of our curve can be almost resolved by a covering then our curve has only one characteristic pair. This algebraic proof is based on the following geometric interpretation of the number of characteristic pairs.

Namely, let us apply a sequence of quadratic transformations to our plane until the total transform of our curve has only normal crossings. In this process, each quadratic transformation is assumed to have its center on the corresponding proper transform of our curve. Concerning the resulting exceptional lines, it can be seen that any one of them is met by at most three others. Moreover, it can be shown that the number of exceptional lines which are met by three others is exactly equal to the number of characteristic pairs of our curve.

The rest of the argument is similar to the argument in my paper "On a Question of Mumford" in the *American Journal.* Briefly speaking, consider the singularity of a cyclic covering of our local plane having our curve as the branch locus, and now, at that singularity, consider the nonrational hidden prime divisors, i.e., the nonrational prime divisors of second kind; they exactly lie over the above mentioned exceptional lines which are met by three others; finally, the genera of these nonrational

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